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Tom's Ten Data Tips – June 2009

History of Mathematics

Mathematics were never invented but rather discovered. It exists in every aspect of our lives, all around us. Humans have been known to use numbers for about 8000 years. Around 3500 BC the first account of cross tabs were kept which Assyrians used for collecting taxes. Maya's used quipus around 2500 BC to keep track of counts. Symbolic mathematics with equations, theorems and proofs began around 500 BC. In the 17th century calculus was invented (independently by Newton and Leibniz) and modern abstract algebra where symbols like x and y denote arbitrary entities stems from the mid 19th century. As this historical account shows, the growth spurt in mathematical development is of relative recent origin.

Unlike most other sciences, advances in mathematics are cumulative rather than disruptive: contributions of earlier mathematicians are rarely discarded in favor of more recent discoveries. Dramatic growth occurred mainly in the 20th century when more mathematics was developed than in 3000 years before. Newer mathematics must pay homage to its antecedents. Although mathematics is used frequently to describe real world phenomena, it exists separate and apart from the real world.

1. Until 500 BC Math Was All About *Numbers*

In ancient Egypt, Babylonia and China, math still revolved solely around numbers. Mathematics could have been equated with arithmetic as we use the term now. Bear in mind that operations we perform on (Arabic!, but also take note of tip# 6) numbers were far from straightforward until then. Math was aimed at measuring, counting and accounting. For engineering, construction and trade these were very valuable skills.

2. Greeks Transformed Math To Include *Shape*

The Greeks turned math into a truly academic discipline. They were mainly concerned with Geometry. For them numbers represented measurements of length. This was why their study of numbers came to a halt after they discovered some lengths didn't correspond to their notion of numbers at the time: they had stumbled on irrational numbers (like π) and didn't have the means yet to deal with those.

Their main representative Euclid wrote a 13-volume tome called "Elements" which reputedly was the second most circulated book (the bible of course #1), and it was certainly the single most important work in Greek mathematics. Although math advanced in several parts of the world (notably China and Arabia), it didn't fundamentally change until the 17th century with the introduction of calculus. Until algebra broke through (medieval era) arithmetic and geometry went on as two relatively unrelated branches of Mathematics.

3. You Can't Square A Circle (Without Irrational Numbers)

One of the most famous problems left by the Greek to posterity was the problem of "squaring a circle." The problem as presented by the Greek was to define a square with the same area as the circle, and to determine the length of its sides. Given the operations available to them, there was no formula available (yet) to express π as a function of numbers.

So many people tried and failed to solve the problem of "squaring a circle" that we now use the expression to describe an insurmountable task.

4. Pythagoras' Theorem Was Known 1000 Years Before His Birth

Probably the most widely known mathematical theorem is Pythagoras': for a right angled triangle the sum of the squares on the shorter sides equal the square of the longest side. Babylonians understood the Pythagorean theorem as early as 1800-1650 BC, more than a thousand years before Pythagoras.

Two tablets show unequivocal evidence Babylonians used Pythagoras' theorem for Geometric calculations and to find solutions to algebraic equations.

5. Calculus Changed Math To A Study Of Motion And Space

In the 17th century both Isaac Newton and Gottfried Wilhelm Leibniz independently invented calculus. In essence, it is the study of motion and change. Now the motion of planets and trajectories of bodies could become object of study, as well as flow of liquids, expansion of gasses, magnetism, electricity, etc. Newton disputed the authenticity of Leibniz' discovery which mushroomed into an ongoing battle that became an issue of national honor with England's scientists pressing the cause in favor of Newton, and most continental scientists siding with Leibniz. Leibniz' notation was superior, but most likely they independently discovered pretty much exactly the same thing. Leibniz introduced the terms differential and integral, and $\delta x/\delta y$ which we still use today.

6. The "Arabic" Number System Was Developed By Hindus

Around 2000 years ago Hindus developed the number system we still use today. It reached its current form around the 6^{th} century. Arabic mathematicians introduced it in the west about a century later. That's why we still call it "the Arabic system" although that description isn't quite true.

Besides this system many others have been, and some are still in use. We have seen the Cuneiform, Etruscan, Mayan, ancient Chinese, Ancient Indian, and Roman. The ancient Chinese and Roman numbbers are still in use. In all systems, the numbers 1-2-3 bear structural similarities, and from there on the numbers always differ. For instance, in Arabic numbers, 1-2-3 were made up of one through three horizontal lines which can still be seen in the way we write 1-2-3. The similarity between 1-2-3 appears to carry some significance with regards to how we process small (up to 3) numbers (almost instantaneously, little cognitive processing) relative to larger quantities.

7. "Arabic" Numbers Made Double-Entry Bookkeeping Possible

The "Arabic system" is far superior in every account. The operations it allows played a pivotal role in developing double entry bookkeeping. Leonardo of Pisa (1180-1250), also known as Fibonacci, introduced the "new" arithmetic using Arabic numbers in his book "Liber Abacci." The operations that became available because of the switch to Arabic numbers enabled double-entry bookkeeping. In 1494 Luca Pacioli published "Summa", now heralded as the first book on accountancy practices.

This, in turn, was adopted first in Venice. Because traders could now dramatically expand their operations and still maintain "control" this subsequently led to their world dominance for a few centuries. Venice dominated until developments in business models, in particular predecessors to the limited liability corporation (by the Dutch) took business expansion to the next level.

8. Probability Helped Deal With Measurement Uncertainty

The study if probability theory didn't really start until the 17^{th} century. The roots of probability theory are commonly ascribed to a series of letters between Pierre de Fermat and Blaise Pascal in 1654. For a long time it was intricately related to gambling. A little later during the Age of Enlightenment statistics was seen as a mathematical way to conduct public policy.

Pascal and Fermat hadn't quite grasped probability to it's full extent, which today seems odd given these were two absolutely brilliant mathematicians. They were calculating with ratios and permutations, and they never fully solved the so-called "gambler's problem" of how to value the odds in an unfinished game of throwing dice, a problem most people would be able to solve today. However, they did pave the way for Kepler, Galileo and Gauss to deal with uncertainty in (astronomical) measurement, leading eventually to what we now call "Gaussian curves." Mathematical statistics in the late 19^{th} century brought together the statistical methods of astronomers with data collection of actuaries.

9. Looking For A Challenge? Eat Your Heart Out!

At the beginning of the 20^{th} century, August 8, 1900, David Hilbert delivered a famous speech about the most difficult, then deemed "unsolvable" mathematical problems. At the moment, a little over 100 years later almost all have been solved. "Solved" sometimes meant proof was delivered that a problem was positively "unsolvable."

The last of this list to be solved was Fermat's theorem which Andrew Wiles thought he solved in 1993. Later it turned out that there was a flaw in his proof, which Wiles together with Richard Taylor corrected in 1994. The Clay Mathematics Institute has awarded a 1 million award for each of seven "millennium problems" http://www.claymath.org/millennium/. Some of the famous intricate problems remaining are the Navier-Stokes equations (see also tip# 10), the Poincaré conjecture, and Riemann's hypothesis.

10. Chaos And Complex Systems Still Largely Escape Us

The rise of computer technology brought new possibilities to the mathematical community. Computers in and of themselves don't have much to offer to mathematicians. However, they have made it possible to perform calculations that were either tedious and time consuming, or outright impossible to perform by humans.

Chaos theory and fractals came to the foreground largely as a result of advances and availability in computer technology. Although a lay person may associate "chaos" with "disorder", chaos theory is wholly deterministic. Nothing disorderly about that. Fractals are related in intricate ways, they appear sometimes as the "art work" of mathematics. Benoit Mendelbrot played a pivotal role in their inclusion in mathematics. Although generating fractals is deterministic, we can't know beforehand which points will be included or not (at least not presently), there is simply no algorithm available to determine that. This adds to their mystery. The fact that many *natural* dynamic systems behave chaotically remains at present one of the mysteries of our universe.

